

FINAL REPORT

**STATISTICAL INFERENCE  
FROM SAMPLED DATA**

ONR Grant N00014-90-J-1175

(January 1, 1990 - March 31, 1998)

Elias Masry

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La Jolla, California 92093

May 1, 1998

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Prepared for

OFFICE OF NAVAL RESEARCH

Surveillance, Communications, and Electronic Combat Division

Arlington, Virginia 22217

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## I. RESEARCH ACCOMPLISHMENTS

Research for ONR Grant "Statistical Inference From Sampled Data" has been concerned in the past 8 years with studies in the general area of statistical signal processing. Contributions were made in a wide range of topics motivated by practical problems in communication systems and digital signal processing. The following is a list of the main topics on which the research was focused:

- A. Inference for Continuous-Time Processes from Sampled Data.
- B. Wavelets, Signal Approximation, and Function Estimation.
- C. Neural Networks and Function Estimation.
- D. Probability Density and Regression Estimation from Noisy Dependent Data.
- E. Nonlinear Time Series.
- F. Local Polynomial Fitting.
- G. Spread Spectrum Communication Systems.
- H. Other Contributions.

The research under this grant resulted in the publication of 32 refereed papers in mathematical, statistical, and engineering journals. In addition, several papers were presented and subsequently published in proceedings of conferences. Copies of these works were routinely sent to the Office of Naval Research, Surveillance, Communications, and Electronic Combat Division. The following is a list of the journal publications under this grant:

1. L. Györfi and E. Masry, "The  $L_1$  and  $L_2$  Strong Consistency of Recursive Kernel Density Estimation From Dependent Samples," *IEEE Trans. Information Theory*, Vol. IT-36 (May 1990), pp. 531-539.
2. E. Masry, "Almost Sure Convergence Analysis of Autoregressive Spectral Estimation in Additive Noise," *IEEE Trans. Information Theory*, Vol. IT-37 (January 1991), pp. 36-42.
3. E. Masry, "Multivariate Probability Density Deconvolution for Stationary Random Processes," *IEEE Trans. Information Theory*, Vol. IT-37 (July 1991), pp. 1105-1115.
4. E. Masry and J. A. Rice, "Gaussian Deconvolution via Differentiation," *Canadian J. Statistics*, Vol. 20, No. 1 (1992), pp. 9-21.
5. E. Masry, "Flicker Noise and the Estimation of the Allan Variance," *IEEE Trans. Information Theory*, Vol. IT-37 (July 1991), pp. 1173-1177.
6. S. Cambanis and E. Masry, "Trapezoidal Stratified Monte Carlo Integration," *SIAM J. Numerical Analysis*, Vol. 29 (February 1992), pp. 284-301.

7. K. S. Lii and E. Masry, "Model Fitting for Continuous-Time Stationary Processes from Discrete-Time Data," *J. Multivariate Analysis*, Vol. 41 (April 1992), pp. 56-79.
8. E. Masry, "Asymptotic Normality for Deconvolution Estimators of Multivariate Densities of Stationary Processes," *J. Multivariate Analysis*, Vol. 44 (January 1993), pp. 47-68.
9. J. Fan and E. Masry, "Multivariate Regression Estimation With Errors-in-Variables: Asymptotic Normality for Mixing Processes," *J. Multivariate Analysis*, Vol. 43 (November 1992), pp. 237-271.
10. C. Lo, E. Masry, and L. B. Milstein, "Design and Analysis of a Fast Frequency-Hopped DBPSK Communication Systems, Part I: System Description and Hop Timing Tracking Loop Analysis," *IEEE Trans. Communications*, Vol. COM-41 (October 1993), pp. 1552-1564.
11. C. Lo, E. Masry, and L. B. Milstein, "Design and Analysis of a Fast Frequency-Hopped DBPSK Communication Systems, Part II: Error Performance in AWGN Plus Partial-Band Noise Jamming," *IEEE Trans. Communications*, Vol. COM-41 (November 1993), pp. 1723-1735.
12. E. Masry, "The Wavelet Transform of Stochastic Processes with Stationary Increments and its Application to Fractional Brownian Motion," *IEEE Trans. Information Theory*, Vol. IT-39 (January 1993), pp. 260-264.
13. E. Masry, "Strong Consistency and Rates for Deconvolution of Multivariate Densities of Stationary Processes," *Stochastic Processes and Applics.*, Vol. 47 (August 1993), pp. 53-74.
14. E. Masry, "Multivariate Regression Estimation with Errors-in-Variables for Stationary Processes," *Nonparametric Statistics*, Vol. 3 (1993), pp. 13-36.
15. S. Cambanis and E. Masry, "Wavelet Approximation of Deterministic and Random Signals: Convergence Properties and Rates," *IEEE Trans. Information Theory*, Vol. IT-40 (July 1994), pp. 1013-1029.
16. K.S. Lii and E. Masry, "Spectral Estimation of Continuous Time Stationary Processes from Random Sampling," *Stochastic Processes and Applics.*, Vol. 52 (August 1994), pp. 39-64.
17. E. Masry, "On the Truncation Error of the Sampling Expansion for Stationary Bandlimited Processes," *IEEE Trans. Signal Processing*, Vol. 42 (October 1994), pp. 2851-2853.
18. E. Masry, "Probability Density Estimation from Dependent Observations using Wavelets orthonormal bases," *Statistics and Probability Letters*, Vol. 21 (1994), pp. 181-194.
19. E. Masry and F. Bullo, "Convergence Analysis of the Sign Algorithm for Adaptive Filtering," *IEEE Trans. Information Theory*, Vol. 41 (March 1995), pp. 489-495.
20. E. Masry and L. Milstein, "Enhanced Signal Interception in the Presence of Interference," *IEEE Trans. Communications*, Vol. COM-43 (March 1995), pp. 1089-1096.

21. K.S. Lii and E. Masry, "On the Selection of Random Sampling Schemes for the Spectral Estimation of Continuous Time Processes," *J. Time Series Analysis*, Vol. 16 (May 1995), pp. 291-311.
22. E. Masry and D. Tjøstheim, "Nonparametric Estimation and Identification of Nonlinear ARCH Time Series," *Econometric Theory*, Vol. 11 (June 1995), pp. 258-289.
23. E. Masry, "Convergence Properties for Wavelet Series Expansions of Fractional Brownian Motion," *Applied and Computational Harmonic Analysis*, Vol. 3 (July 1996), pp. 239-254.
24. D. Modha and E. Masry, "Rate of Convergence in Density Estimation Using Neural Networks," *Neural Computation*, Vol. 8 (July 1996), pp. 1102-1122.
25. E. Masry, "Multivariate Local Polynomial Regression for Time Series: Uniform Strong Consistency and Rates," *J. Time Series Analysis*, Vol. 17 (November 1996), pp. 571-599.
26. D. Modha and E. Masry, "Minimum Complexity Regression Estimation With Weakly Dependent Observations," *IEEE Trans. Information Theory*, Vol. 42 (November 1996), pp. 2133-2145.
27. E. Masry "Multivariate Regression Estimation: Local Polynomial Fitting for Time Series," *Stochastic Processes and Their Applications*, Vol. 65 (December 1996), pp. 81-101.
28. E. Masry, "Polynomial Interpolation and Prediction of Continuous-Time Processes From Random Samples," *IEEE Trans. Information Theory*, Vol. 43 (March 1997), pp. 776-783.
29. E. Masry and D. Tjøstheim, "Additive Nonlinear ARX Time Series and Projection Estimates," *Econometric Theory*, Vol. 13 (April 1997), pp. 214-252.
30. E. Masry and J. Fan, "Local Polynomial Estimation of Regression Functions for Mixing Processes," *Scandinavian J. of Statistics*, Vol. 24 (June 1997), pp. 165-179.
31. E. Masry, "Multivariate Probability Density Estimation by Wavelet Methods: Strong Consistency and Rates for Stationary Time Series," *Stochastic Processes and Their Applics.*, Vol. 67 (May 1997), pp. 177-193.
32. E. Masry, "Spectral and Multivariate Probability Density Estimation of Continuous-Time Stationary Processes From Randomly Sampled Data," invited paper, in *Applications of Time Series Analysis in Astronomy and Meteorology*, T. Subba Rao, M.B. Priestley, and O. Lessi, Eds., London: Chapman & Hall, 1997. pp. 89-102.

## II. DESCRIPTIVE SUMMARY OF PROBLEMS AND RESULTS

A descriptive summary of the research problems studied under this grant and the nature of the results obtained is now presented for each of the principal areas listed in Section I.

### A. Inference for Continuous-Time Processes from Sampled Data

Research in this area focused on three problems: a) spectral estimation, b) prediction and interpolation, and c) Monte Carlo integration. Our contributions consist of the papers [1][2][3][4][5][6].

#### a) Spectral Estimation.

Let  $X = \{X(t), -\infty < t < \infty\}$  be a continuous-time stationary process,  $\{\tau_k\}$  be the sampling instants, and  $\{X(\tau_k)\}$  be the discrete-time observation process. We are interested in estimating the statistical structure of the continuous-time process  $\{X(t), -\infty < t < \infty\}$  from a finite set of discrete-time observations  $\{X(\tau_k), \tau_k\}_{k=1}^n$ . Particular functions of interest are the family of finite-dimensional distributions and densities of the process  $X$ , the correlation function  $C(t)$ , and the spectral density  $\phi(\lambda)$  of the process  $X$ . Clearly if the sampling instants  $\{\tau_k\}$  are equally-spaced, consistent estimates of the joint densities of the process  $X$  from the observations  $\{X(\tau_k), \tau_k\}$  is not feasible; similarly, if the process  $X$  is not bandlimited, consistent estimates of  $C(t)$  and  $\phi(\lambda)$  from equally-spaced observations is not possible (due to aliasing) unless the sampling rate is allowed to diverge to infinity. Our main goal was to identify appropriate nonequally-spaced sampling schemes with a finite average sampling rate which would allow the consistent estimation of these functions as the number of observations  $n$  tends to infinity. Such sampling schemes are called alias-free.

We considered both parametric [1] and nonparametric [2] approaches: Let  $X = \{X(t), -\infty < t < \infty\}$  be a continuous-time stationary process with spectral density  $\phi_X(\lambda; \underline{\theta})$  where  $\underline{\theta}$  is a vector of unknown parameters. For example,  $\underline{\theta}$  could be the parameters of an autoregressive moving average continuous-time process. Let  $\{\tau_k\}$  be a stationary point process on the real line which is independent of  $X$ . We considered in [1] the identifiability and estimation of  $\underline{\theta}$  from the discrete-time observations  $\{X(\tau_k), \tau_k\}_{k=1}^n$ . We established the consistency and the joint asymptotic normality of appropriate estimates  $\hat{\underline{\theta}}_n$ . In [2] we provided a complete statistical analysis of suitable estimates of the (nonparametric) spectral density function  $\phi(\lambda)$ . We identified broad classes of nonequally-spaced sampling schemes  $\{\tau_k\}$  with finite mean sampling rates and formulated suitable spectral estimates  $\hat{\phi}(\lambda)$  on the basis of the discrete-time observations  $\{X(\tau_k), \tau_k\}_{k=1}^n$ . We established the quadratic-mean consistency and asymptotic normality of such estimates as the number of observations  $n$  tends to infinity. It was shown that the spectral estimates are consistent for all positive values of the average sampling rate. No such results could exist for equally-spaced samples. In [3] we provided practical guidelines for the selection of appropriate alias-free sampling schemes  $\{\tau_k\}$  for the estimation of broadband and narrowband spectra. In [4] a review of available results for the estimation of the covariance function  $C(t)$ , spectral density  $\phi(\lambda)$ , and the multivariate probability densities  $f(x_1, \dots, x_m; t_1, \dots, t_m)$  of continuous-time processes from discrete-time observations is presented.

### b) Interpolation and Prediction.

One of the classical problems in signal estimation is the recovery of a continuous-time signal  $f(t)$  from a sequence of its samples  $\{f(t_j)\}$ . The well-known Shannon's sampling theorem interpolates a bandlimited signal from equally-spaced samples taken at the Nyquist rate. In [5] we considered the polynomial interpolation and prediction of second order continuous-time random processes from a finite number of randomly sampled observations  $\{X(t_j)\}$ ; the sampling instants  $\{t_j\}$  constitute a general stationary point process on the real line which is independent of  $X$ . We establish in [5] upper bounds on the mean-square interpolation and prediction errors, valid for all mean sampling rates  $\beta > 0$ , and explored their dependence on  $\beta$ , on the number of samples, and on the irregularity of the sampling instants.

### c) Monte Carlo Integration.

In [6] weighted integrals of random processes are approximated by the trapezoidal rule based on a stratified and symmetrized random sample of size  $n$ . We established the rate of convergence to zero of the mean-square integral approximation error as the sample size increases indefinitely. For random processes which are twice mean-square continuously differentiable it is shown that the rate is  $n^{-5}$ , just as without a random component. For random processes which are a bit more than once, but not twice, mean-square continuously differentiable the rate is shown to be  $n^{-4}$ . In both cases the asymptotic constant is also determined. This work extends the results of Haber [7][8] for deterministic functions to Monte Carlo integration of random processes.

## **B. Wavelets, Signal Approximation, and Function Estimation**

Wavelet analysis of signals is a rapidly developing area of research. It has found applications in a wide variety of fields: in digital signal processing because of its connection to multirate filtering; in image processing because of applications in image representation and compression and multiscale edge detection; in numerical analysis because of applications to partial differential equations; and in signal processing (and mathematical physics) because of the time-frequency (phase-space) localization properties of the wavelet transform. See the books by Meyer [9], Daubechies [10], and Chui [11] and the special issue of the IEEE Transactions on Information Theory (March 1992).

Our contributions in this area consist of the papers [12][13][14][15][16]. We focused on two aspects of wavelet analysis: a) Approximation of functions and random processes via orthonormal wavelet bases, and b) the use of wavelet-based methods for the estimation of multivariate probability density functions.

### a) Approximation Results.

In [12] we developed an  $n^{\text{th}}$  order expansion for the wavelet approximation error  $e_m^2$  at resolution  $2^{-m}$  for functions  $f \in L_2(-\infty, \infty)$  having a given degree of smoothness (say  $n$  derivatives). Specifically, the approximation of  $f$  at resolution  $2^{-m}$  is given by

$$f_m(t) = \sum_{k=-\infty}^{\infty} a_{m,k} \phi_{m,k}(t), \quad a_{m,k} = \int_{-\infty}^{\infty} f(x) \phi_{m,k}(x) dx$$

where  $\phi_{m,k}(x) = 2^{-m/2} \phi(2^m x - k)$  and  $\phi(x)$  is the scale function. The exact rate of convergence and



asymptotic constants were determined and their dependence on  $f$  and on the scale function  $\phi$  are obtained: If  $f$  has  $n$  continuous and integrable derivatives, then we established that

$$e_m^2 \triangleq \int_{-\infty}^{\infty} [f(t) - f_m(t)]^2 dt = \frac{C_2}{2^{2m}} + \dots + \frac{C_{2[n/2]}}{2^{2[n/2]m}} + o\left(\frac{1}{2^{nm}}\right)$$

where  $C_0 = 0$  and for  $k \geq 1$ ,

$$C_{2k} = \frac{(-1)^{k+1}}{(2k)!} M_{2k} \int_{-\infty}^{\infty} [f^{(k)}(t)]^2 dt, \quad M_{2k} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (u-v)^{2k} \phi(u)\phi(v) du dv.$$

Note that the constants  $M_{2k}$  depend only on the scale function  $\phi$ . When  $n \geq 2$  and  $M_2 \neq 0$ , then

$$2^{2m} e_m^2 \rightarrow \frac{1}{2} M_2 \int_{-\infty}^{\infty} [f^{(1)}(t)]^2 dt \quad \text{as } m \rightarrow \infty$$

and this rate cannot be improved by additional differentiability of  $f$  beyond second-order. However, faster rates of convergence are possible if certain centered moments of the scale function  $\phi$  vanish. These results complement and refine those of Mallat [17, Theorem 3] and Meyer [9] which lead to an upper bound on the rate of convergence of the error  $e_m^2$ .

In the same work [12] we also considered the wavelet approximation at resolution  $2^{-m}$  of stationary and nonstationary second-order random processes. All stationary and most nonstationary second-order processes do not have sample paths in  $L_2(-\infty, \infty)$  and thus they do not fit the standard framework of  $L_2(-\infty, \infty)$  wavelet representation. However, with probability one, the sample functions of mean-square continuous stationary and nonstationary random processes are square integrable over every finite interval. We therefore considered in [12] the wavelet approximation of such processes, at resolution  $2^{-m}$ , over a finite interval, say  $[0, T]$ ,

$$\hat{X}_m(t) = \sum_{k=-\infty}^{\infty} a_{m,k} \phi_{m,k}(t)$$

where the series converges in  $L_2[0, T]$  with probability one and

$$a_{m,k} = \int_{-\infty}^{\infty} X(t) \phi_{m,k}(t) dt,$$

and we provided an  $n^{\text{th}}$  order expansion for the mean integrated squared approximation error,

$$e_{m,T}^2 = E \int_0^T [X(t) - \hat{X}_m(t)]^2 dt$$

as  $m \rightarrow \infty$  under very mild smoothness assumptions on the correlation function  $R(t, s) = E[X(t)X(s)]$ . The precise rate of convergence as well as the asymptotic constant were determined in [12]. These results complement and refine some results in Cohen et al. [18] where an upper bound on the rate of convergence of the error  $e_{m,T}^2$  is given under a condition on the decay at infinity of the spectral density.

In [13] we considered the approximation of Fractional Brownian motion (FBM) by a wavelet-based representation at resolution  $2^{-m}$ . Note that FBM has no quadratic-mean derivatives and hence the results we obtained in [12] are not applicable. Fractional Brownian motion is a widely used model in signal processing. We obtained asymptotic expressions for the mean integrated squared approximation error over finite intervals.

#### b) Density Estimation.

Probability density estimation is a fundamental problem in statistical data analysis. Recently, wavelet methods were introduced for the estimation of probability density functions. The first papers by Doukhan and Léon [19], Kerkyacharian and Picard [20], and Walter [21] dealt with linear wavelet estimators in i.i.d. setting and established upper bounds on convergence in the mean  $L_p$  norm. Leblanc [22] extended the work of Kerkyacharian and Picard [20] to dependent observations again establishing upper bounds in the mean  $L_p$  norm estimation error. Donoho et al. [23] were the first to introduce nonlinear wavelet estimates using thresholded empirical wavelet coefficients and established minimax results over a range of densities belonging to the Besov space  $B_{s,p,q}$ ,  $s > 0$ ,  $1 \leq p \leq \infty$ ,  $1 \leq q \leq \infty$  and a range of global mean  $L_{p'}$  error measures with  $1 \leq p' \leq \infty$ . For certain values of  $p$  and  $p'$ , they showed that linear estimates have a suboptimal rates of convergence.

Our contributions in this area consist of the papers [14][15]. In [14] we established precise mean  $L_2$ -norm results (rate of convergence and the value of the asymptotic constant) for densities in the Sobolev space  $H_2^s$ ,  $s > 0$ , for dependent observations using linear wavelet density estimates. This improved upon the earlier works where only an upper bound on the error was obtained. In [15] we considered the estimation of the multivariate probability density functions  $f(\underline{x}) = f(x_1, \dots, x_d)$ ,  $d \geq 1$ , of the random process  $\{X_i\}$  using linear wavelet methods. We established the strong consistency of such estimates along with rates of convergence which are uniform over compact subsets of  $R^d$ . The multivariate density function  $f$  is assumed to belong to the Besov space  $B_{s,p,q}$ . It is shown that these uniform almost sure rates coincide with the best attainable rates and thus nonlinear thresholded estimates (highly hyped in the literature) provide no improvement in this case. It should be noted that the primary reason for the claimed performance advantage of nonlinear density estimates in Donoho et al. [23] is that the adopted performance measure there is mean integrated error. This advantage disappears when a sup-norm error is used as the performance measure. It should also be noted that for rich classes of functions such as the Besov class  $B_{s,p,q}$  (where functions can have irregular behavior), a measure of performance based on an integrated error is unsuitable: Large estimation errors can occur over small sets in  $R^d$  which contribute little to the integrated error; a sup-norm performance measure is clearly much more meaningful. Also, in practice, estimates are based on a single realization and thus an almost sure convergence rates are desirable. Consequently an almost sure sup-norm performance measure is the appropriate choice for estimating density functions in the Besov space  $B_{s,p,q}$  and the results we obtained in [15] imply that nonlinear thresholded density estimates have no real advantage over linear wavelet-based density estimates.

In [16] we considered the wavelet transform and wavelet coefficients of random processes with wide-sense stationary increments. Their covariance functions and spectral distributions are determined.

These results extend the work of Flandrin [24] for the special case of fractional Brownian motion.

### C. Neural Networks and Function Estimation

The research in this area focused on probability density and regression functions estimation using single hidden layer sigmoidal neural networks. Our contribution consists of two papers [25][26].

In [25] we considered the estimation of a probability density functions  $f(\underline{x})$ ,  $\underline{x} \in R^d$ . we constructed minimum complexity density estimators  $\hat{f}_n(\underline{x})$  based on  $n$  i.i.d. observations  $\{\underline{X}_i\}_{i=1}^n$ . For densities  $f$  belonging to the exponential family, we establish a rate of convergence for the expected Hellinger distance,

$$E \int \left[ \sqrt{\hat{f}_n(\underline{x})} - \sqrt{f(\underline{x})} \right]^2 d\underline{x} = O\left(\left(\frac{\log n}{n}\right)^{1/2}\right).$$

The important point here is that the rate of convergence is *independent* of the dimension  $d$  in sharp contrast to the standard methods (kernel, histogram, nearest neighbor density estimates) which suffer from the "curse of dimensionality" whereby the rate of convergence becomes progressively slower as the dimension  $d$  increases.

In [26] we considered multivariate regression estimation for dependent data using neural networks. Let  $\{Y_i, \underline{X}_i\}$  be a bivariate stationary process and define the regression function

$$r(\underline{x}) \triangleq E[Y_1 | \underline{X}_1 = \underline{x}], \quad \underline{x} \in R^d.$$

Minimum complexity estimation of  $r(\underline{x})$  was considered previously by Barron [27] in the context of i.i.d. data. In [26] we extended Barron's result to the case of dependent data: For strongly mixing processes with geometric decay, we showed that the mean integrated squared error of estimates  $\hat{r}(\underline{x})$ , based on the observations  $\{Y_i, \underline{X}_i\}_{i=1}^n$ , satisfies

$$E \int_{R^d} [\hat{r}_n(\underline{x}) - r(\underline{x})]^2 dP_{\underline{X}}(\underline{x}) = O\left(\left(\frac{\log n}{n^{1/2}}\right)^{1/2}\right).$$

Again note that the rate of convergence is independent of the dimension  $d$  in contrast to the curse of dimensionality exhibited by standard kernel regression estimates.

### D. Probability Density and Regression Estimation from Noisy Dependent Data

Probability density and regression estimation play a central role in communication theory, pattern recognition and classification [28]-[29]. We considered the problem of estimating multivariate probability densities and regression functions in the presence of additive noise. Our contributions in this area are represented by papers [30][31][32][33][34][35].

#### a) Density Estimation

Let  $\{X_i^o\}_{i=-\infty}^{\infty}$  be a stationary process and for each integer  $p \geq 1$  let  $f^o(\underline{x}; p) = f^o(x_1, \dots, x_p; p)$  be the joint probability density function of the random variables  $X_1^o, \dots, X_p^o$ . Consider the deconvolution

problem

$$X_i = X_i^\circ + \varepsilon_i$$

where the (noise) process  $\{\varepsilon_i\}_{i=-\infty}^{\infty}$  consists of i.i.d. random variables with known density  $h(x)$ , independent of the process  $\{X_i^\circ\}_{i=-\infty}^{\infty}$ . Given the noisy observations  $\{X_i\}_{i=1}^n$  one desires to estimate the multivariate density  $f^\circ(\underline{x}; p)$ . This is clearly a probability density deconvolution problem. It arises in biological studies (Medgyessy [36], Mendelsohn and Rice [37]), communication theory (Wise et al [38], Snyder et al [39]), and applied physics and analytical chemistry (Jones and Misell [40], Harder and Galan [41]). The special case of i.i.d. observations ( $X_i^\circ$  are i.i.d. random variables and  $p = 1$ ) received considerable attention in the literature with contributions by Carroll and Hall [42], Liu and Taylor [43], Stefanski and Carroll [44], Zhang [45], and Fan [46].

In a series of papers [30] [32] [33] we extended the above works in the following directions:

- The process  $\{X_i^\circ\}_{i=-\infty}^{\infty}$  is a stationary mixing process.
- Estimation of all the joint probability density functions of the process  $\{X_i^\circ\}_{i=-\infty}^{\infty}$  is considered.

In [30] we established the quadratic-mean convergence of deconvolution estimators  $\hat{f}_n^\circ(\underline{x}; p)$  along with rates of convergence for processes satisfying a variety of mixing conditions. In [33] we established sharp rates of almost sure convergence for the estimators  $\hat{f}_n^\circ(\underline{x}; p)$ . In [32] we derived the asymptotic normality of the estimators  $\hat{f}_n^\circ(\underline{x}; p)$ . In all three papers the dependence of the rate of convergence on the smoothness of the noise density  $h(x)$  is investigated. Generally, the smoother the noise density, the slower is the rate of convergence. Thus, for example, when the noise is Gaussian, which is super smooth, the rates of convergence are quite slow.

#### b) Regression Estimation

Let  $\{X_i^\circ\}_{i=-\infty}^{\infty}$  and  $\{Y_i\}_{i=-\infty}^{\infty}$  be jointly stationary processes and  $\{\varepsilon_i\}$  a sequence of i.i.d. random variables, independent of the processes  $\{X_i^\circ\}_{i=-\infty}^{\infty}$  and  $\{Y_i\}_{i=-\infty}^{\infty}$ , with a common probability density  $h(x)$ . Put

$$X_i = X_i^\circ + \varepsilon_i, \quad i = 0, \pm 1, \dots$$

Let  $G(x)$  be an arbitrary function on the real line, not necessarily bounded or continuous, and assume that  $E|G(Y_1)| < \infty$ . Define the multivariate regression function by

$$m(\underline{x}; p) = E[G(Y_p) | \underline{X}_0^\circ = \underline{x}]$$

where

$$\underline{X}_0^\circ = (X_1^\circ, \dots, X_p^\circ).$$

Given the noisy observations  $\{X_j, Y_j\}_{j=1}^n$  one desires to estimate  $m(\underline{x}; p)$ . The problem of regression with errors-in-variables arises, for example, in medical and epidemiologic studies where risk factors are partially available, see Prentice [47] and Whittemore and Keller [48].

In [34][35] we provided a thorough analysis of the multivariate regression problem with errors in variables for dependent data. In [35] we obtained sharp rates of almost sure convergence for the

deconvolution estimates of  $m(\underline{x}; p)$  for strongly mixing processes. In [34] we established the asymptotic normality of these estimates for a variety of mixing processes. These results significantly extend the earlier works by Fan and Truong [49] who examined the special case of i.i.d. observations (the variables  $\{X_i^o, Y_i\}$  are i.i.d.,  $p = 1$  and  $G(x) = x$ ).

### E. Nonlinear Time Series

Nonlinear time series is a very active area of research in general [50] [51] and in econometrics in particular [52] [53]. Among the models considered in the literature are the autoregressive conditional heteroscedastic (ARCH) model [54] which had a substantial impact on modeling of econometric time series and the autoregressive with exogenous variables (ARX) model. Our contributions in this area consist of the papers [55][56].

The ARCH model has the representation

$$X_t = g_1(X_{t-q}, \dots, X_{t-1}) + g_2(X_{t-q}, \dots, X_{t-1}) e_t$$

where  $g_2 \geq 0$  and the innovations  $\{e_t\}$  are i.i.d. random variables with zero mean and variance  $\sigma^2$ . A primary research focus has been the identification/estimation of the nonlinear functions  $g_1$  and  $g_2$  from data. When  $g_1$  and  $g_2$  are specified in a parametric form (e.g.  $g_1$  being linear and  $g_2$  being quadratic), estimates of their parameters were considered in [54]. In [55] we provided a thorough analysis of the convergence properties of nonparametric estimates of  $g_1$  and of  $\sigma^2 g_2^2$  including sharp rates of almost sure convergence and asymptotic normality.

In [56] we considered the identification/estimation of the functional structure of the bivariate nonlinear ARX time series

$$Y_t = g_3(Y_{t-q}, \dots, Y_{t-1}) + g_4(X_{t-p}, \dots, X_t) + e_t$$

$$X_t = g_5(X_{t-p}, \dots, X_{t-1}) + \varepsilon_t.$$

where  $\{e_t\}$  and  $\{\varepsilon_t\}$  are independent series each consisting of zero mean i.i.d. variables with variances  $\sigma_e^2$  and  $\sigma_\varepsilon^2$  respectively. There is an extensive literature on parametric ARX models whereby the functional structure of the  $g_i$  is known except for finite number of parameters and a rigorous theory is available (see, for example, Pötscher and Prucha [57][58]). No rigorous theory was available in a nonparametric setting. Using the projection idea of Auestad and Tjøstheim [59] we provided in [56] a comprehensive analysis of suitable estimates of the functions  $g_3$  and  $g_4$  including rates of weak convergence and asymptotic normality. An important consequence of these result is the dimension reduction in the sense that the rate of convergence for the estimate of  $g_3$  involves only its dimension  $q$  rather than  $p + q$  obtained normally by standard regression estimates. Similarly for  $g_4$ .

### F. Local Polynomial Fitting

Local polynomial fitting is a new methodology for estimation of regression functions which dramatically outperforms the classical kernel approach of Nadaraya [60] - Watson [61]. The local polynomial fitting approach was introduced originally by Stone [62] and studied by Cleveland [63], Tsybakov [64], Fan [65] [66] and many others. Local polynomial fitting has significant advantages over

the Nadaraya-Watson regression estimator: For local linear fitting it has been shown to reduce the bias (see Chu and Marron [67] and Fan [65]); it adapts automatically to the boundary of design points (see Fan and Gijbels [68], Ruppert and Wand [69] — No boundary modification is required. It is superior to the Nadaraya-Watson estimator in the context of estimating the derivatives of the regression function (see Fan and Gijbels [68] and Ruppert and Wand [69]). All these works consider i.i.d. setting and either linear or quadratic polynomial fitting.

During the current grant period we embarked on a major project to extend these works to more general and useful situations: Let  $\{Y_i, X_i\}$  be jointly stationary processes and let the regression function  $m(x_1, \dots, x_p)$  be defined by

$$m(x_1, \dots, x_p) \triangleq E[Y_p | X_1 = x_1, \dots, X_p = x_p].$$

For example, when  $Y_i = X_{i+r}$ , the regression function  $m$  gives the best  $r$ -step predictor of the process  $\{X_i\}$ . In a series of papers [70][71][72] we formulated and established the convergence properties of local polynomial fitting of *arbitrary* order  $q \geq 1$  (not just  $q = 1$  and  $q = 2$ ). Moreover, we allowed the processes  $\{Y_i, X_i\}$  to be general dependent processes rather than i.i.d. pairs. In [70] we established sharp rates of almost sure convergence for the local polynomial estimates of the regression function  $m(x_1, \dots, x_p)$  and all its partial derivatives up to a total order  $q$ . In [71] we established the joint asymptotic normality of these estimates along with asymptotic expressions for their bias and variance.

### G. Spread Spectrum Communication Systems

Spread Spectrum communication systems offer immunity against narrow-band interference [73]. There are various techniques for enhancing this immunity including interference suppression filters and frequency hopping methods.

In [74][75] we provided a comprehensive analysis (and design) of fast frequency-hopped spread spectrum communication system employing differential binary phase shifted keying (DBPSK) modulation and differentially coherent modulation. The receiver utilizes a hop timing tracking loop to "lock" the hop clock. In [74] the performance characteristics of the tracking loop is studied. In [75] we obtained the bit error probability of the overall system when it is operating under additive white Gaussian noise plus partial-band jamming. Our modeling and results extend those in the literature [76] where the hop timing tracking errors are ignored.

In [77] we considered the problem of intercepting a direct sequence spread spectrum signal which is embedded in both narrowband interference and additive white Gaussian noise. We employed a compression receiver the heart of which is a surface acoustic wave (SAW) device for interference rejection. A central limit theorem is established for the receiver's output which forms the basis for analyzing the performance of the receiver. Numerical results are presented to illustrate the receiver's operating characteristics

### H. Other Contributions

Additional contributions [78][79][80][81][82] were made in a broad range of topics. We briefly summarize the contributions of the more important ones [78][80][82].

In [78] we considered recursive probability density estimation for vector-valued processes  $\{\underline{X}_j\}$  from dependent samples  $\{\underline{X}_j\}_{j=1}^n$ . We established rates of almost sure convergence for the global  $L_p$  error

$$e_n^{(p)} \triangleq \int_{R^d} |\hat{f}_n(\underline{x}) - f(\underline{x})|^p d\underline{x}$$

for  $p = 1, 2$ . We obtained results for both ergodic processes and mixing processes. In addition, the notion and properties of Hilbert space-valued mixingales are developed and strong laws of large numbers are given. The results on Hilbert space-valued mixingales extend those of McLeish [83] who dealt with real-valued mixingales. The results on density estimates extend the point-wise convergence results of Masry [84] and Tran [85] which were obtained under considerably stronger conditions.

In the area of parametric spectral estimation we considered in [80] the problem of estimating the spectral density  $\phi(\lambda)$  of an autoregressive (AR(p)) stationary process

$$X_j - \sum_{i=1}^p a_i X_{j-i} = \varepsilon_j$$

from a finite set of *noisy* observations.

$$Y_j = X_j + W_j.$$

This is a classical and practically important problem in signal processing. A modified spectral estimator based on the high-order Yule-Walker equations was considered. Sharp rates of almost sure convergence are established for the estimates of the autoregressive parameters, the innovation variance, the additive noise variance, and the spectral density  $\phi(\lambda)$  of the AR process  $\{X_i\}$ . No almost sure rates of convergence were previously available. The work supplements the asymptotic normality result obtained earlier by Gingras and Masry [86] and the work of Pagano [87] where nonlinear regression method is used. The advantage of using the (linear) high-order Yule-Walker equations is the simplicity of implementation.

Adaptive linear estimation methods based on the principle of steepest descent and its variations have been applied to a wide range of problems such as filtering, noise canceling, line enhancement, antenna processing, and interference suppression. The well known LMS algorithm has been extensively studied in the literature. One of the most popular and widely used algorithms is the sign algorithm whose recursive equations are as follows: The updated equation for the vector  $\mathbf{h}(j)$  of the estimated filter's coefficients at iteration  $j$  is given by

$$\mathbf{h}(j+1) = \mathbf{h}(j) + \mu \mathbf{x}(j) \operatorname{sgn}[e(j)], \quad j = 1, 2, \dots$$

where  $\mathbf{x}(j)$  is the data at iteration  $j$ ,  $e(j)$  is the error in estimating the desired signal  $d(j)$  using the data vector  $\mathbf{x}(j)$ ,

$$e(j) = d(j) - \mathbf{h}^T(j) \mathbf{x}(j),$$

and  $\mu$  is the adaptation size. Here  $\mathbf{h}(j)$  and  $\mathbf{x}(j)$  are column vectors with dimension  $N$ . The popularity of the sign algorithm is due to its simplicity of implementation in that only the polarity of the error  $e(j)$  is used to update the estimate. One is concerned with the convergence of the sign algorithm as  $j \rightarrow \infty$ : If  $\mathbf{h}_{opt}$  is the optimal Wiener-Hopf filter with corresponding minimum mean-square error  $\varepsilon_{min}^2$ , then one seeks the convergence properties of the deviation error

$$\mathbf{v}(j) = \mathbf{h}(j) - \mathbf{h}_{opt}$$

and of the signal's estimation error  $e(j)$  as  $j \rightarrow \infty$ . For a fixed step size  $\mu > 0$ , rigorous convergence analysis was first given in Gersho [88]; heuristic results were later given in [89][90][91]. Asymptotic results, as the step size  $\mu \rightarrow 0$  were established in [92].

In [82] we established asymptotic time-averaged convergence for the mean-square deviation error  $E[\|\mathbf{v}(j)\|^2]$  and for the signal estimation error  $E[e^2(j)]$  under the assumption of i.i.d. Gaussian data. Specifically we proved that for any initial weight vector  $\mathbf{h}(1)$  and for any positive step size  $\mu$  we have

$$\limsup_{n \rightarrow \infty} \frac{1}{n} \sum_{j=1}^n E[\|\mathbf{v}(j)\|^2] \leq C_1 \mu + C_2 \mu^2.$$

Moreover, we showed that

$$\frac{1}{n} \sum_{j=1}^n E[e^2(j)] = \varepsilon_{\min}^2 + \frac{1}{n} \sum_{j=1}^n \varepsilon^2(j)$$

with

$$\limsup_{n \rightarrow \infty} \frac{1}{n} \sum_{j=1}^n \varepsilon^2(j) \leq C_3 \mu + C_4 \mu^2$$

where the constants  $C_i$  are specified in [82]. Thus the time-averaged mean-square deviation error and the excess signal estimation error are proportional to the step size  $\mu$  which is similar to the behavior of the LMS algorithm.

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